SMT359/Specimen Equations booklet

$$\operatorname{div}\mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \int_S \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\int_V \rho\, \mathrm{d}V$$

$$\operatorname{div}\mathbf{B} = 0 \qquad \int_S \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$$

$$\operatorname{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint_C \mathbf{E}\cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \mathbf{B}\cdot \mathrm{d}\mathbf{S}$$

$$\operatorname{curl}\mathbf{B} = \mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial \mathbf{E}}{\partial t} \qquad \oint_C \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \int_S \left(\mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial \mathbf{E}}{\partial t}\right)\cdot \mathrm{d}\mathbf{S}$$

$$\operatorname{div}\mathbf{D} = \rho_\mathrm{f} \qquad \int_S \mathbf{D}\cdot \mathrm{d}\mathbf{S} = \int_V \rho_\mathrm{f}\, \mathrm{d}V$$

$$\operatorname{div}\mathbf{B} = 0 \qquad \int_S \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$$

$$\operatorname{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint_C \mathbf{E}\cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \mathbf{B}\cdot \mathrm{d}\mathbf{S}$$

$$\operatorname{curl}\mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_C \mathbf{H}\cdot \mathrm{d}\mathbf{l} = \int_S \left(\mathbf{J}_\mathrm{f} + \frac{\partial \mathbf{D}}{\partial t}\right)\cdot \mathrm{d}\mathbf{S}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}\mathbf{J} = 0 \qquad \mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}, \quad \mathbf{B} = \mu\mu_0\mathbf{H}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\,\widehat{\mathbf{r}}_{12} \qquad \nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\mathbf{F}_{12} = \frac{\mu_0}{4\pi}\frac{I_1\delta\mathbf{l}_1\times(I_2\,\delta\mathbf{l}_2\times\widehat{\mathbf{r}}_{12})}{r_{12}^2} \qquad \mathbf{N} = \mathbf{E}_\mathrm{phys}\times\mathbf{H}_\mathrm{phys}$$

$$\mathbf{E} = -\operatorname{grad}V, \qquad V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2}\mathbf{E}\cdot\mathrm{d}\mathbf{l}$$

Fundamental constants

speed of light in vacuum c	$3.00\times10^{8}\,\mathrm{m\,s^{-1}}$
permittivity of free space ε_0	$8.85\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ (or $\mathrm{F\,m^{-1}}$)
permeability of free space μ_0	$4\pi\times10^{-7}\,\mathrm{N\,A^{-2}}$ (or $\mathrm{H\,m^{-1}}$)
electron charge $-e$	$-1.60\times10^{-19}\,\mathrm{C}$
electron mass	$9.11\times10^{-31}\,\mathrm{kg}$
proton mass	$1.67\times10^{-27}\,\mathrm{kg}$
Boltzmann's constant k_B	$1.38\times10^{-23}\,\mathrm{J\,K^{-1}}$
Avogadro's number N_A	$6.02\times10^{23}\,\mathrm{mol^{-1}}$

Unit conversions

N	$\equiv \mathrm{kg\,m\,s^{-2}}$	
J	$\equiv \mathrm{N\,m}$	$\equiv \mathrm{kg\,m^2\,s^{-2}}$
W	$\equiv \mathrm{J\,s^{-1}}$	$\equiv \mathrm{A\,V} \equiv \mathrm{kg\,m^2\,s^{-3}}$
Ω	$\equiv \mathrm{V\,A^{-1}}$	$\equiv \mathrm{kg\,m^2\,s^{-3}\,A^{-2}}$
C	$\equiv \mathrm{A\,s}$	
F	$\equiv \mathrm{C\,V^{-1}}$	$\equiv \mathrm{kg^{-1}\,m^{-2}\,s^4\,A^2}$
T	$\equiv \mathrm{N\,A^{-1}\,m^{-1}}$	$\equiv \mathrm{V\,s\,m^{-2}} \equiv \mathrm{kg\,s^{-2}\,A^{-1}}$
H	$\equiv \mathrm{T\,m^2\,A^{-1}}$	$\equiv \mathrm{V\,s\,A^{-1}} \equiv \mathrm{kg\,m^2\,s^{-2}\,A^{-2}}$

2.1 PT/TEX
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Theorems

$$\int_{\mathbf{r_1}}^{\mathbf{r_2}} \operatorname{grad} f \cdot \mathrm{d}\mathbf{l} = f(\mathbf{r_2}) - f(\mathbf{r_1})$$

$$\int_V \operatorname{div} \mathbf{F} \, \mathrm{d}V = \int_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$$

$$\int_S \operatorname{curl} \mathbf{F} \cdot \mathrm{d}\mathbf{S} = \oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{l}$$

Vector and vector calculus identities

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$

$$\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \operatorname{grad} f$$

$$\operatorname{div}(\operatorname{grad} f) = \nabla^2 f$$

$$\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$$

$$\operatorname{curl}(\operatorname{grad} f) = \mathbf{0}$$

$$\operatorname{curl}(\operatorname{curl} \mathbf{F}) = \operatorname{grad}(\operatorname{div} \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$\operatorname{div}(\mathbf{F} \times \mathbf{G}) = (\operatorname{curl} \mathbf{F}) \cdot \mathbf{G} - \mathbf{F} \cdot (\operatorname{curl} \mathbf{G})$$

Various integrals

$$\int x^n \, \mathrm{d}x = \frac{1}{n+1} x^{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln x + C$$

$$\int \sin(ax) \, \mathrm{d}x = -\frac{1}{a} \cos(ax) + C$$

$$\int \cos(ax) \, \mathrm{d}x = \frac{1}{a} \sin(ax) + C$$

$$\int \exp(ax) \, \mathrm{d}x = \frac{1}{a} \exp(ax) + C$$

$$\int \ln(ax) \, \mathrm{d}x = x \ln(ax) - x + C$$

$$\int x \, \mathrm{e}^{-ax} \, \mathrm{d}x = -\frac{1}{a^2}(1 + ax) \, \mathrm{e}^{-ax} + C$$

$$\int_0^{2\pi} \sin^2 \theta \, \mathrm{d}\theta = \int_0^{2\pi} \cos^2 \theta \, \mathrm{d}\theta = \pi$$

$$\int x^2 \, \mathrm{e}^{-ax} \, \mathrm{d}x = -\frac{1}{a^3}(2 + 2ax + a^2x^2) \, \mathrm{e}^{-ax} + C$$

$$\langle \sin^2 \theta \rangle \equiv \frac{1}{2\pi} \int_0^{2\pi} \sin^2 \theta \, \mathrm{d}\theta = \frac{1}{2}$$

$$\int \frac{1}{(a^2 + x^2)^{1/2}} \, \mathrm{d}x = \ln((a^2 + x^2)^{1/2} + x) + C$$

$$\langle \cos^2 \theta \rangle \equiv \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta \, \mathrm{d}\theta = \frac{1}{2}$$

$$\int \frac{1}{(a^2 + x^2)^{3/2}} \, \mathrm{d}x = \frac{x}{a^2 \sqrt{a^2 + x^2}} + C$$

$$\int \cos^n \theta \, \sin \theta \, \mathrm{d}\theta = -\frac{\cos^{n+1}(\theta)}{n+1} + C$$

$$\int_{-\infty}^{\infty} \frac{1}{(1 + x^2)^{3/2}} \, \mathrm{d}x = 2$$

$$\int_0^{2\pi} \cos \theta \, \sin \theta \, \mathrm{d}\theta = 0$$

Differential operations for a scalar field f and a vector field $\mathbf{F}$

Cartesian coordinates (x, y, z)

$$\operatorname{grad} f = \frac{\partial f}{\partial x}\mathbf{e}_x + \frac{\partial f}{\partial y}\mathbf{e}_y + \frac{\partial f}{\partial z}\mathbf{e}_z$$

$$\operatorname{div} \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{e}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{e}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{e}_z$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical coordinates (r, ϕ, z)

$$\operatorname{grad} f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \phi}\mathbf{e}_\phi + \frac{\partial f}{\partial z}\mathbf{e}_z$$

$$\operatorname{div} \mathbf{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\operatorname{curl} \mathbf{F} = \frac{1}{r}\begin{vmatrix} \mathbf{e}_r & r\mathbf{e}_\phi & \mathbf{e}_z \\ \partial/\partial r & \partial/\partial \phi & \partial/\partial z \\ F_r & rF_\phi & F_z \end{vmatrix}$$

$$= \left(\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\mathbf{e}_r + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\mathbf{e}_\phi + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial \phi}\right)\mathbf{e}_z$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical coordinates (r, θ, ϕ)

$$\operatorname{grad} f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\mathbf{e}_\phi$$

$$\operatorname{div} \mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\operatorname{curl} \mathbf{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{e}_r & r\mathbf{e}_\theta & r\sin\theta\,\mathbf{e}_\phi \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ F_r & rF_\theta & r\sin\theta\, F_\phi \end{vmatrix}$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\sin\theta\, F_\phi) - \frac{\partial F_\theta}{\partial \phi}\right)\mathbf{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right)\mathbf{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right)\mathbf{e}_\phi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Miscellaneous formulae

$\mathbf{J} = nq\mathbf{v}$

$\delta\mathbf{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,\delta\mathbf{l}' \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$

$\delta\mathbf{F} = I\,\delta\mathrm{l}\times\mathbf{B}$

$\mathbf{m} = |I|\,\Delta\mathbf{S}$

$\mathbf{B} = \dfrac{\mu_0 I}{2\pi r}\,\mathbf{e}_\phi$

$\mathbf{B} = \mu_0 nI\,\mathbf{e}_z$

$\mathbf{p} = q\mathbf{d}$

$V(\mathbf{r}) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\mathbf{p}\cdot\widehat{\mathbf{r}}}{r^2}$

$C = Q/V$

$U = \frac{1}{2}CV^2$

$V_{\mathrm{emf}} = \oint_C (\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\mathrm{d}\mathbf{l}$

$V = IR$

$U = -\mathbf{p}\cdot\mathbf{E}$

$\mathbf{\Gamma} = \mathbf{p}\times\mathbf{E}$

$\mathbf{P} = n\langle\mathbf{p}\rangle = \chi_E\varepsilon_0\mathbf{E}$

$\sigma_{\mathrm{b}} = \mathbf{P}\cdot\widehat{\mathbf{n}}$

$\rho_{\mathrm{b}} = -\mathrm{div}\,\mathbf{P}$

$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$

$\varepsilon = 1+\chi_E$

$E_{1\|} = E_{2\|};\quad D_{2\perp} - D_{1\perp} = \sigma_{\mathrm{f}}$

$U = -\mathbf{m}\cdot\mathbf{B}$

$\mathbf{\Gamma} = \mathbf{m}\times\mathbf{B}$

$\mathbf{M} = n\langle\mathbf{m}\rangle = \chi_B\mathbf{B}/\mu_0$

$\mathbf{i}_{\mathrm{b}} = \mathbf{M}\times\widehat{\mathbf{n}}$

$\mathbf{J}_{\mathrm{b}} = \mathrm{curl}\,\mathbf{M}$

$\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$

$\mu = (1-\chi_B)^{-1}$

$H_{2\|} - H_{1\|} = i_{\mathrm{s}};\quad B_{1\perp} = B_{2\perp}$

$B = \dfrac{\mu_0 I}{4\pi d}\left[\sin\alpha_2 - \sin\alpha_1\right]$

$\omega_{\mathrm{c}} = \dfrac{|q|B}{m}$

$r_{\mathrm{c}} = \dfrac{mv_\perp}{|q|B}$

$\mathbf{J} = \sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B})$

$\Phi = \displaystyle\int_S \mathbf{B}\cdot\mathrm{d}\mathbf{S}$

$V_{\mathrm{emf}} = -\dfrac{\mathrm{d}\Phi}{\mathrm{d}t}$

$M_{21} = \dfrac{\mathrm{d}\Phi_{21}}{\mathrm{d}I_1}$

$L = \dfrac{\mathrm{d}\Phi}{\mathrm{d}I}$

$U = \frac{1}{2}\displaystyle\sum_{i=1}^{n} q_i V_i$

$$U = \frac{1}{2}\int_\tau \rho(\mathbf{r})V(\mathbf{r})\,\mathrm{d}\tau + \frac{1}{2}\int_S \sigma(\mathbf{r})V(\mathbf{r})\,\mathrm{d}S$$

$u = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}$

$I = \dfrac{V_{\mathrm{s}}}{R}\exp\left(-\dfrac{t}{RC}\right)$

$I = \dfrac{V_{\mathrm{s}}}{R}\left[1 - \exp\left(-\dfrac{Rt}{L}\right)\right]$

$U = \frac{1}{2}LI^2$

$u = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$

$\omega_{\mathrm{n}} = 1/\sqrt{LC}$

$\mathrm{curl}\,\mathbf{J}_{\mathrm{s}} = -\dfrac{n_{\mathrm{s}}e^2}{m}\mathbf{B}$

$\dfrac{\partial\mathbf{J}_{\mathrm{s}}}{\partial t} = \dfrac{n_{\mathrm{s}}e^2}{m}\mathbf{E}$

$\nabla^2\mathbf{B} = \dfrac{1}{\lambda^2}\mathbf{B}$

$\lambda = \left(\dfrac{m}{\mu_0 n_{\mathrm{s}} e^2}\right)^{1/2}$

$\rho' = \gamma\left(\rho - \dfrac{v}{c^2}J_x\right)$

$J'_x = \gamma(J_x - v\rho);\quad J'_y = J_y;\quad J'_z = J_z$

$\mathbf{E}'_\| = \mathbf{E}_\|;\quad \mathbf{E}'_\perp = \gamma(\mathbf{E}_\perp + \mathbf{v}\times\mathbf{B}_\perp)$

$\mathbf{B}'_\| = \mathbf{B}_\|;\quad \mathbf{B}'_\perp = \gamma\left(\mathbf{B}_\perp - \dfrac{\mathbf{v}\times\mathbf{E}_\perp}{c^2}\right)$

$v_{\mathrm{phase}} = \omega/k$

$c = 1/\sqrt{\varepsilon_0\mu_0}$

$\nabla^2\mathbf{E} = \varepsilon_0\mu_0\dfrac{\partial^2\mathbf{E}}{\partial t^2}$

$\mathbf{B} = \dfrac{1}{c}\,\widehat{\mathbf{k}}\times\mathbf{E}$

$\exp\mathrm{i}\theta = \cos\theta + \mathrm{i}\sin\theta$

$\mathbf{N} = \mathbf{E}_{\mathrm{phys}}\times\mathbf{H}_{\mathrm{phys}}$

$\overline{\mathbf{N}} = \frac{1}{2}\varepsilon\varepsilon_0 E_0^2\dfrac{c}{n}\,\widehat{\mathbf{k}}$

$v = 1/\sqrt{\varepsilon\varepsilon_0\mu\mu_0}$

$n = c/v$

$n_1\sin\theta_1 = n_2\sin\theta_2$

$\theta_{\mathrm{B}} = \tan^{-1}(n_2/n_1)$

$v_{\mathrm{group}} = \dfrac{\mathrm{d}\omega}{\mathrm{d}k}$

$\delta = \sqrt{2/\mu_0\sigma\omega}$

$\nabla^2\mathbf{E} = \mu_0\sigma\dfrac{\partial\mathbf{E}}{\partial t}$

$k_{\mathrm{gw}}^2 = k_0^2 - \left(\dfrac{m\pi}{a}\right)^2 - \left(\dfrac{n\pi}{b}\right)^2$

$\omega_{\mathrm{p}} = \sqrt{\dfrac{n_{\mathrm{e}}e^2}{m\varepsilon_0}}$

$\omega = \sqrt{\omega_{\mathrm{p}}^2 + k^2c^2}$

$\varepsilon_{\mathrm{eff}}(\omega) = 1 - \dfrac{\omega_{\mathrm{p}}^2}{\omega^2}$